1. (21%) In the following, find $\frac{dy}{dx}$

(a) $4 \cos x \sin y = 1$  
(b) $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$  
(c) $y = \int_1^x \sin t \, dt$

2. (28%) Evaluate the following integral

(a) $\int \frac{x^2}{\sqrt{1-x^2}} \, dx$  
(b) $\int_0^{\pi/2} x \sin^2 x \cos x \, dx$  
(c) $\int \frac{1}{\sqrt{x^2 + 16}} \, dx$  
(d) $\int e^x \sin x \, dx$

3. (8%) Find the limit if it exists or show that the limit does not exist.

$$\lim_{(x,y) \to (0,0)} \frac{6x^3 y}{2x^4 + y^4}$$

4. (8%) Find the shortest distance from the point $(1,0,-2)$ to the plane $x + 2y + z = 4$.

5. (10%) The region $D$ enclosed by the curves $y^2 = x$ and $x = 2y$ is rotated about the $y$-axis. Find the volume of the resulting solid.

6. (28%) Examine the following series for convergence:

(a) $\sum_{k=1}^{\infty} ke^{-k^2}$,  
(b) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k^2 + 1}\right)^3$,  
(c) $\sum_{n=1}^{\infty} (\sqrt{1+n^2} - n)$,  
(d) $\sum_{n=1}^{\infty} n! e^{-n}$.

7. (12%) Prove that there is a solution of the equation

$$\frac{1}{\sqrt{x + x^2}} + x^2 - 2x = 0, \quad x > 0.$$
8. (15%) If \( f \) has a finite third derivative \( f'' \) in \([a,b]\) and if
\[
f(a) = f'(a) = f(b) = f'(b) = 0.
\]
Prove that \( f''(c) = 0 \) for some \( c \) in \((a,b)\).

9. (20%) Discuss whether the following sets are open or closed. Justify your answers.
   
   (a) \( S = \{(x, y) \in \mathbb{R}^2 \mid x \text{ is rational}\} \).
   
   (b) \( S = \{(x, y) \in \mathbb{R}^2 \mid xy > 1\} \).
   
   (c) \( S = \{(-1)^n + 1/m \mid m, n \in \mathbb{N}\} \).
   
   (d) \( S = \{2^{-n} + 5^{-m} \mid m, n \in \mathbb{N}\} \).