1. Find the following limit
   (a) \( \lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h} \)  
   (b) \( \lim_{x \to \infty} \left( e^x + x \right)^{\frac{1}{x}} \)  
   (12 points)

2. In the following, find \( \frac{dy}{dx} \)  
   (18 points)
   (a) \( y \sin x^2 = x \sin y^2 \)  
   (b) \( y = (\sin x)^{\ln x} \)  
   (c) \( y = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}} dt \)

3. Evaluate the following integral (20 points)
   (a) \( \int_{\text{e}}^{\text{e}^4} \frac{1}{x \sqrt{\ln x}} dx \)  
   (b) \( \int e^x \sqrt{1 + e^x} dx \)  
   (c) \( \int \frac{\tan^3 x}{\cos x} dx \)  
   (d) \( \int_{0}^{1} \frac{1}{x^2 - 6x + 5} dx \)

4. The region \( D \) enclosed by the curves \( y = x \) and \( y = x^2 \) is rotated about the line \( y = 2 \). Find the volume of the resulting solid. (10 points)

5. Determine whether the series is convergent or divergent. (15 points)
   (a) \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \)  
   (b) \( \sum_{n=1}^{\infty} \left( \frac{n}{n+1} \right)^n \)  
   (C) \( \sum_{n=1}^{\infty} (-1)^n \sin \left( \frac{\pi}{n} \right) \)

6. Suppose \( A, B \) are open sets in \( \mathbb{R} \) and \( F, G \) are closed sets in \( \mathbb{R} \).
   (a) Prove that \( A \cup B \) and \( A \cap B \) are open. (10 points)
   (b) Prove that \( F \cap G \) is closed. (5 points)
7. Suppose \( \{x_n\} \) is a sequence of real numbers such that

\[
(a) |x_n - x_{n+1}| \leq \frac{1}{n+5} \quad \text{prove or disprove that } \{x_n\} \text{ converges.} \quad (10 \text{ points})
\]

\[
(b) |x_n - x_{n+1}| \leq \frac{1}{n^2} \quad \text{prove or disprove that } \{x_n\} \text{ converges.} \quad (10 \text{ points})
\]

8. (a) Find an open cover of \([0,1)\) with no subcover in \(\mathbb{R}\). (10 points)

(b) Is \( \left\{ 0, 1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots \right\} \) compact in \(\mathbb{R}\)? Justify your answer. (10 points)

9. (a) Let \( \{A_k\} \) be a sequence of compact nonempty sets in \(\mathbb{R}\) such that \(A_{k+1} \subset A_k\) for all \(k \in \mathbb{N}\). Prove that \( \bigcap_{k \in \mathbb{N}} A_k \neq \emptyset \). (10 points)

(b) Find an example of \( \{A_k\} \) in \(\mathbb{R}\) such that \(A_{k+1} \subset A_k\) for all \(k \in \mathbb{N}\) but \( \bigcap_{k \in \mathbb{N}} A_k = \emptyset \). (10 points)